MAT 161 - Answers to Sample Final

1) a) $1/2$ b) $-\infty$

2) a) $f'(x) = \dfrac{x^3 \cdot \frac{1}{x} - (\ln x)\cdot 3x^2}{x^6} = \dfrac{1 - 3\ln x}{x^4}$

b) $g'(x) = e^{\sin^{-1}x}\cdot(-\csc 5x \cot 5x)\cdot 5 + (\csc 5x)\cdot e^{\sin^{-1}x}\cdot\dfrac{1}{\sqrt{1-x^2}}$

c) $h'(x) = 5\sin^4(\cos^3 x)\cdot\cos(\cos^3 x)\cdot 3\cos^2 x\cdot(-\sin x)$

3) a) $-\frac{1}{10}(x^5+1)^{-2} + C$ (let $u = x^5+1$)

b) $\frac{1}{8}$ (let $u = \tan x$)

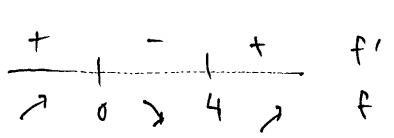
4) a) 0 b) 2 c) $(-\infty, -1) \cup (1, \infty)$

Note that $A'(x) = (x^2-1)^{1/3}$

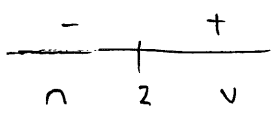
5) a) (i) does not exist (ii) 1 (iii) 3 (iv) does not exist

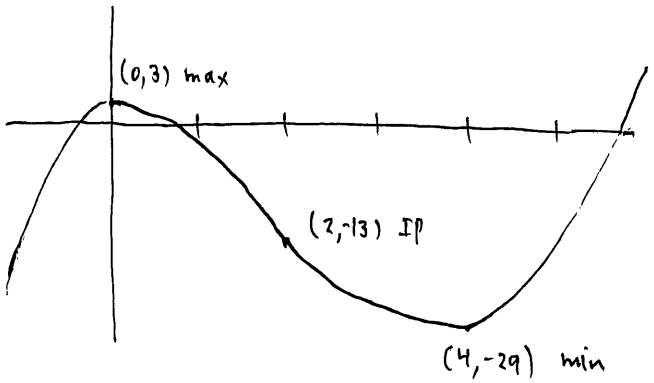
b) $x = 1, 3, 5$

6) $f'(x) = 3x(x-4)$



$f''(x) = 6(x-2)$





7) $f'(x) = \lim_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \dfrac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$

$= \lim_{h \to 0} \dfrac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \dfrac{1}{\sqrt{x+h} + \sqrt{x}} = \dfrac{1}{2\sqrt{x}}$

8) $\dfrac{dy}{dx} = \dfrac{y^3 \sin x - 1}{3y^2 \cos x - \frac{4}{y^2}} \rightsquigarrow y = -\frac{1}{11}x + 2$

9) $\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt} \Rightarrow \dfrac{dr}{dt} = \dfrac{2}{\pi}$ m/hour

10) $v(t) = 6t^2 + 10$

$\Rightarrow s(3) - s(0) = \int_0^3 (6t^2 + 10)\, dt \Rightarrow s(3) = 104$ m

11) $R_4 = \frac{1}{2} \ln 1.5 + \frac{1}{2} \ln 2 + \frac{1}{2} \ln 2.5 + \frac{1}{2} \ln 3$

12) Maximize $V = \pi r^2 h$ subject to $\pi r^2 + 2\pi r h = 1000$.

$h = \dfrac{1000 - \pi r^2}{2\pi r}$

$\Rightarrow V(r) = \pi r^2 \left(\dfrac{1000 - \pi r^2}{2\pi r} \right) = \dfrac{r}{2}(1000 - \pi r^2)$

$= 500r - \frac{1}{2}\pi r^3$

$V'(r) = 500 - \frac{3}{2}\pi r^2 = 0$

$\Rightarrow r = \left(\dfrac{1000}{3\pi} \right)^{1/2}$

$V''(r) = -3\pi r < 0 \quad \cap$ max

Largest volume is $V\left(\left(\dfrac{1000}{3\pi} \right)^{1/2} \right)$.