

IDENTICAL VANISHING OF COEFFICIENTS IN THE SERIES EXPANSION OF ETA QUOTIENTS, MODULO 4, 9 AND 25

TIM HUBER, JAMES MCLAUGHLIN, AND DONGXI YE

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Please see the paper [insert link to published version of paper at journal webs site later] for a description of what the numbers in the table mean.

Table 1: The count of zero coefficients modulo 25 (columns 3 -7) and the count of zero coefficients (columns 8 - 12) in the first 15000 terms in the series expansion of $F(q)(f_1^5/f_5)^j$, $0 \leq j \leq 4$.

n	$F(q)$	0	1	2	3	4	0	1	2	3	4
1	f_1	14810	3199	3199	3199	3199	14800	22	0	0	0
2	f_1^2	9139	7811	7811	7811	7811	9132	6113	0	0	0
3	$\frac{f_1^2}{f_2}$	14887	3476	3476	3476	3476	14877	4	0	0	0
4	f_2	14869	4320	4320	4320	4320	14859	31	0	0	0
5	$\frac{f_2^2}{f_1}$	14837	3087	3087	3087	3087	14827	2	0	0	0
6	$\frac{f_2^6}{f_1^2}$	6579	6579	8613	6579	6579	6123	7	0	0	0
7	f_3	14895	5956	5956	5956	5956	14885	189	0	0	0
8	$f_1 f_3$	6721	4466	4466	4466	4466	6717	22	0	0	0
9	$\frac{f_1^2 f_3}{f_2}$	11845	2453	2453	2453	2453	11837	2	0	0	0
10	$f_2 f_3$	8197	1707	1707	1707	1707	8190	1	0	0	0
11	$\frac{f_2^2 f_3}{f_1}$	10841	2452	2452	2452	2452	10836	13	0	0	0
12	$\frac{f_2 f_3^2}{f_1}$	8443	1651	1651	1651	1651	8438	5	0	0	0
13	$\frac{f_3^3}{f_1}$	8928	6821	6821	6821	6821	8924	0	0	0	0
14	$f_1 f_3^3$	6617	6617	8689	6617	6617	6123	0	3	0	0

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15	$\frac{f_2^2}{f_4}$	14923	4463	4463	4463	4463	14913	25	0	0	0
16	$\frac{f_2^3}{f_1 f_4}$	14810	2924	2924	2924	2924	14800	12	0	0	0
17	$\frac{f_2^5}{f_1^2 f_4}$	11691	2619	2619	2619	2619	11684	11	0	0	0
18	$\frac{f_2^3 f_3}{f_1 f_4}$	8266	1687	1687	1687	1687	8260	1	0	0	0
19	$\frac{f_2^4 f_3}{f_1^2 f_4}$	6168	1229	1229	1229	1229	6161	1	0	0	0
20	f_4	14910	4868	4868	4868	4868	14900	37	0	0	0
21	$\frac{f_1 f_4}{f_2}$	14837	3095	3095	3095	3095	14827	1	0	0	0
22	$f_2 f_4$	12633	6218	6218	6218	6218	12625	26	0	0	0
23	$\frac{f_2^2 f_4}{f_1}$	9008	2187	2187	2187	2187	9003	6	0	0	0
24	$f_3 f_4$	9311	1865	1865	1865	1865	9303	0	0	0	0
25	$\frac{f_2 f_3 f_4}{f_1}$	9991	2018	2018	2018	2018	9984	0	0	0	0
26	$\frac{f_4^2}{f_2}$	14887	3707	3707	3707	3707	14878	2	0	0	0
27	$\frac{f_1 f_4^2}{f_2}$	9008	7650	7650	7650	7650	9003	4004	0	0	0
28	$\frac{f_3 f_4^2}{f_2}$	11968	3346	3346	3346	3346	11961	78	0	0	0
29	$\frac{1}{f_1 f_5}$	4715	4715	4715	5426	4715	0	0	2051	2	0
30	$\frac{f_1 f_2}{f_4 f_5}$	6581	8633	6581	6581	6581	3	1	0	0	0
31	$\frac{f_4^2}{f_1^2 f_2 f_5}$	2741	2741	2741	3454	2741	0	1	0	3	0
32	f_5	14921	3068	3068	3068	3068	14911	23	0	0	0
33	$\frac{f_5}{f_1}$	6804	6804	6804	8570	6804	0	6408	0	4	0
34	$f_1 f_5$	12168	9207	9207	9207	9207	12161	7887	5661	0	0
35	$\frac{f_1^2 f_5}{f_2}$	11310	7722	7722	7722	7722	11301	6371	0	0	0
36	$f_2 f_5$	11059	7279	7279	7279	7279	11052	5503	0	0	0
37	$\frac{f_2^2 f_5}{f_1}$	12948	10900	10900	10900	10900	12939	10081	0	0	0
38	$\frac{f_5}{f_1^2 f_3}$	4876	11652	4876	4876	4876	0	11645	2	0	0
39	$f_3 f_5$	10977	4880	4880	4880	4880	10969	0	0	0	0
40	$\frac{f_2 f_3 f_5}{f_1}$	10551	3540	3540	3540	3540	10545	179	0	0	0
41	$\frac{f_2^2 f_5}{f_4}$	11599	3865	3865	3865	3865	11591	256	0	0	0
42	$\frac{f_2^3 f_5}{f_1 f_4}$	11806	5071	5071	5071	5071	11797	476	0	0	0
43	$f_4 f_5$	11411	5209	5209	5209	5209	11404	476	0	0	0
44	$\frac{f_1 f_4 f_5}{f_2}$	12878	4440	4440	4440	4440	12870	257	0	0	0
45	$\frac{f_4^2 f_5}{f_2}$	11395	7462	7462	7462	7462	11386	5501	0	0	0
46	$\frac{f_4^2 f_5}{f_1^2 f_2}$	7539	7539	7539	9103	7539	0	7124	1	7	0
47	$\frac{f_2^2 f_4^2 f_5}{f_1}$	9640	10995	9640	9640	9640	9256	10987	0	0	0
48	$\frac{f_5^2}{f_1^2 f_3}$	13693	7571	7571	7571	7571	13684	6123	6123	0	0
49	$\frac{f_5^4}{f_1}$	2490	2490	2490	3476	2490	0	1252	1	1250	0
50	$\frac{f_4^2 f_5^4}{f_1^2 f_2}$	2693	2693	2693	3617	2693	0	1250	2	1253	0
51	$\frac{f_2^3 f_3^3}{f_1^2 f_6}$	7180	1494	1494	1494	1494	7174	0	0	0	0
52	$\frac{f_2^3}{f_6}$	13251	6923	6923	6923	6923	13244	11	1	0	0

53	$\frac{f_2^2 f_3}{f_6}$	7103	1379	1379	1379	1379	7097	0	0	0	0
54	$\frac{f_2^2 f_3}{f_1 f_6}$	14887	3152	3152	3152	3152	14877	7	0	0	0
55	$\frac{f_2^3 f_3}{f_1 f_6}$	7920	2191	2191	2191	2191	7914	0	0	0	0
56	$\frac{f_3^2}{f_6}$	14939	3299	3299	3299	3299	14929	4	0	0	0
57	$\frac{f_2 f_3^2}{f_6}$	9865	2884	2884	2884	2884	9859	6	0	0	0
58	$\frac{f_2 f_3^2}{f_1 f_6}$	14810	3054	3054	3054	3054	14800	2	0	0	0
59	$\frac{f_2 f_3^3}{f_1 f_6}$	8371	3634	3634	3634	3634	8365	4	0	0	0
60	$\frac{f_2^2 f_3 f_4}{f_1 f_6}$	10787	2125	2125	2125	2125	10781	2	0	0	0
61	$\frac{f_3^2 f_4}{f_6}$	12041	2379	2379	2379	2379	12033	1	0	0	0
62	$\frac{f_2^2 f_3 f_5}{f_1 f_6}$	10568	2357	2357	2357	2357	10559	33	0	0	0
63	$\frac{f_3^2 f_5}{f_6}$	11656	4855	4855	4855	4855	11648	502	0	0	0
64	$\frac{f_2 f_3^2 f_5}{f_1 f_6}$	12237	5777	5777	5777	5777	12228	846	0	0	0
65	f_6	14928	3395	3395	3395	3395	14919	16	0	0	0
66	$f_1 f_6$	8194	1696	1696	1696	1696	8189	0	0	0	0
67	$\frac{f_1^2 f_6}{f_2}$	10841	2430	2430	2430	2430	10836	7	0	0	0
68	$f_2 f_6$	10737	2721	2721	2721	2721	10731	0	0	0	0
69	$\frac{f_1^2 f_6}{f_3}$	7125	1405	1405	1405	1405	7119	0	0	0	0
70	$\frac{f_1^2 f_6}{f_2 f_3}$	14837	2941	2941	2941	2941	14827	7	0	0	0
71	$f_3 f_6$	13375	2740	2740	2740	2740	13367	5	0	0	0
72	$\frac{f_2^5 f_6}{f_1^2 f_4^2}$	10841	2187	2187	2187	2187	10836	2	0	0	0
73	$\frac{f_2^3 f_6}{f_1 f_4}$	8194	1638	1638	1638	1638	8189	0	0	0	0
74	$\frac{f_1 f_2^3 f_6}{f_3 f_4}$	8288	1653	1653	1653	1653	8282	0	0	0	0
75	$\frac{f_2^2 f_3 f_6}{f_1 f_4}$	10844	2114	2114	2114	2114	10839	2	0	0	0
76	$f_4 f_6$	11495	2311	2311	2311	2311	11487	3	0	0	0
77	$\frac{f_1^2 f_4 f_6}{f_2 f_3}$	8016	1601	1601	1601	1601	8011	1	0	0	0
78	$\frac{f_3 f_4 f_6}{f_2}$	7687	1520	1520	1520	1520	7681	1	0	0	0
79	$\frac{f_4^2 f_6}{f_2}$	12844	2653	2653	2653	2653	12835	5	0	0	0
80	$f_5 f_6$	11425	2455	2455	2455	2455	11418	8	0	0	0
81	$\frac{f_5 f_6}{f_1^2 f_2 f_3}$	1944	8371	1944	1944	1944	0	8365	2	0	0
82	$\frac{f_1^2 f_5 f_6}{f_2 f_3}$	11370	2932	2932	2932	2932	11361	8	0	0	0
83	$\frac{f_2^2}{f_6}$	13894	3033	3033	3033	3033	13885	9	0	0	0
84	$\frac{f_1 f_6^2}{f_2}$	8427	1649	1649	1649	1649	8419	1	0	0	0
85	$\frac{f_6^2}{f_3}$	14910	3113	3113	3113	3113	14900	2	0	0	0
86	$\frac{f_1 f_6^2}{f_3}$	8437	2612	2612	2612	2612	8430	3	0	0	0
87	$\frac{f_1 f_6^2}{f_2 f_3}$	14868	3465	3465	3465	3465	14858	3	0	0	0
88	$\frac{f_2 f_6^2}{f_3}$	9438	2619	2619	2619	2619	9428	8	0	0	0
89	$\frac{f_2^2 f_6^2}{f_1 f_4}$	8427	1644	1644	1644	1644	8419	2	0	0	0

90	$\frac{f_2^2 f_6^2}{f_3 f_4}$	11968	2264	2264	2264	2264	11961	2	0	0	0
91	$\frac{f_4 f_6^2}{f_2}$	11625	2362	2362	2362	2362	11617	6	0	0	0
92	$\frac{f_4 f_6^2}{f_3}$	9828	1899	1899	1899	1899	9822	1	0	0	0
93	$\frac{f_1 f_4 f_6^2}{f_2 f_3}$	13301	2584	2584	2584	2584	13292	5	0	0	0
94	$\frac{f_5 f_6^2}{f_3}$	11288	5509	5509	5509	5509	11281	845	0	0	0
95	$\frac{f_1 f_5 f_6^2}{f_2 f_3}$	10883	4694	4694	4694	4694	10874	501	0	0	0
96	$\frac{f_2^2 f_3^3}{f_1 f_6^2}$	10329	4184	4184	4184	4184	10323	4	0	0	0
97	$\frac{f_1 f_6^3}{f_2 f_3}$	9312	2580	2580	2580	2580	9305	2	0	0	0
98	$\frac{f_5 f_6^3}{f_1^2 f_2^2 f_3^2}$	1508	7168	1508	1508	1508	0	7159	0	0	0
99	$\frac{f_1 f_6^4}{f_2 f_3^2}$	9603	2082	2082	2082	2082	9595	5	0	0	0
100	$f_1 f_7$	12528	5913	5913	5913	5913	12522	19	0	0	0
101	f_7^2	14037	2834	2834	2834	2834	14028	8	0	0	0
102	$\frac{f_1^6}{f_2^2 f_8^2}$	11926	2751	2751	2751	2751	11918	3	0	0	0
103	$\frac{f_4^4}{f_2 f_8}$	12633	2962	2962	2962	2962	12625	7	0	0	0
104	$\frac{f_4^3 f_6}{f_2 f_8}$	11523	2293	2293	2293	2293	11516	1	0	0	0
105	$\frac{f_1 f_4^3 f_6^2}{f_2^2 f_3 f_8}$	8437	1678	1678	1678	1678	8430	2	0	0	0
106	f_8	14939	4666	4666	4666	4666	14929	75	1	0	0
107	$f_1 f_8$	9315	1921	1921	1921	1921	9311	2	0	0	0
108	$\frac{f_5^2 f_8}{f_1^2 f_4^2}$	12924	3439	3439	3439	3439	12917	112	0	0	0
109	$\frac{f_2 f_8}{f_4}$	14887	3024	3024	3024	3024	14878	2	0	0	0
110	$\frac{f_2^2 f_8}{f_1 f_4}$	9315	1859	1859	1859	1859	9311	1	0	0	0
111	$\frac{f_1 f_4 f_8}{f_2 f_3 f_8}$	11384	2266	2266	2266	2266	11375	2	0	0	0
112	$\frac{f_2^2 f_3 f_8}{f_1 f_6}$	12148	2499	2499	2499	2499	12142	2	0	0	0
113	$\frac{f_2 f_3^2 f_8}{f_1 f_6}$	9571	1942	1942	1942	1942	9564	1	0	0	0
114	$f_6 f_8$	12079	2453	2453	2453	2453	12069	5	0	0	0
115	$\frac{f_1^2 f_6 f_8}{f_3 f_4}$	9183	1893	1893	1893	1893	9179	1	0	0	0
116	f_9	14943	4816	4816	4816	4816	14933	81	0	0	0
117	$f_1 f_9$	8688	1898	1898	1898	1898	8681	2	0	0	0
118	$\frac{f_2^3 f_9}{f_1 f_4}$	8699	1738	1738	1738	1738	8692	2	0	0	0
119	$\frac{f_2 f_3^2 f_9}{f_1 f_6}$	8685	1784	1784	1784	1784	8678	5	0	0	0
120	f_{10}	14947	3359	3359	3359	3359	14937	55	0	0	0
121	$\frac{f_1^2 f_{10}}{f_2}$	11552	3700	3700	3700	3700	11544	20	0	0	0
122	$\frac{f_2^2 f_{10}}{f_1^2 f_4^2}$	11552	2957	2957	2957	2957	11544	28	0	0	0
123	$\frac{f_2^2 f_{10}}{f_4}$	13070	6215	6215	6215	6215	13060	21	0	0	0
124	$\frac{f_1 f_4 f_6^8}{f_2^2 f_3^3 f_{12}^3}$	8371	1689	1689	1689	1689	8365	0	0	0	0
125	$\frac{f_4 f_6^5}{f_3^2 f_{12}^2}$	12041	2327	2327	2327	2327	12033	0	0	0	0
126	$\frac{f_1 f_4 f_6^5}{f_2^2 f_3 f_{12}^2}$	8288	1647	1647	1647	1647	8282	0	0	0	0

127	$\frac{f_6^6}{f_3^2 f_{12}^2}$	12881	2548	2548	2548	2548	12874	3	0	0	0
128	$\frac{f_4 f_5 f_6^6}{f_1^2 f_2^3 f_3^3 f_{12}^2}$	1747	8266	1747	1747	1747	0	8260	1	0	0
129	$\frac{f_1 f_4 f_6^5 f_8}{f_2^2 f_3^2 f_{12}^2}$	9571	1894	1894	1894	1894	9564	1	0	0	0
130	$\frac{f_6^5 f_9}{f_3^2 f_{12}^2}$	13895	3047	3047	3047	3047	13887	28	0	0	0
131	$\frac{f_1 f_4 f_6^3 f_9}{f_2^2 f_3^2 f_{12}^2}$	8725	1743	1743	1743	1743	8718	2	0	0	0
132	$\frac{f_4^3 f_6}{f_2 f_{12}}$	11326	2227	2227	2227	2227	11318	4	0	0	0
133	$\frac{f_4^4 f_6}{f_2^2 f_{12}}$	12633	2533	2533	2533	2533	12625	9	0	0	0
134	$\frac{f_2 f_3 f_6^2}{f_1 f_{12}}$	8288	1550	1550	1550	1550	8282	2	0	0	0
135	$\frac{f_1 f_6^3}{f_3 f_{12}}$	8266	1643	1643	1643	1643	8260	2	0	0	0
136	$\frac{f_2^3 f_3^3}{f_1 f_3 f_{12}}$	10639	2098	2098	2098	2098	10634	4	0	0	0
137	$\frac{f_2 f_6}{f_1 f_3 f_4 f_{12}}$	6721	1378	1378	1378	1378	6717	1	0	0	0
138	$\frac{f_4 f_6}{f_2 f_{12}}$	11576	2498	2498	2498	2498	11569	4	0	0	0
139	$\frac{f_1 f_4 f_6^3}{f_2 f_3 f_{12}}$	10841	2177	2177	2177	2177	10836	3	0	0	0
140	$\frac{f_4^3 f_6}{f_2 f_3 f_{12}}$	11968	2407	2407	2407	2407	11961	2	0	0	0
141	$\frac{f_6}{f_3 f_{12}}$	13375	2628	2628	2628	2628	13367	3	0	0	0
142	$\frac{f_1^2 f_4 f_6^4}{f_2^2 f_3^2 f_{12}}$	9593	2266	2266	2266	2266	9589	2	0	0	0
143	$\frac{f_6 f_8}{f_{12}}$	13443	2689	2689	2689	2689	13434	7	1	0	0
144	$\frac{f_1 f_4 f_6^2 f_8}{f_2 f_3 f_{12}}$	12148	2871	2871	2871	2871	12142	2	0	0	0
145	$\frac{f_2 f_6^2 f_8}{f_3 f_4 f_{12}}$	11384	2271	2271	2271	2271	11375	1	0	1	0
146	$\frac{f_4 f_6^2 f_9}{f_3 f_{12}}$	9307	1892	1892	1892	1892	9298	0	0	0	0
147	$\frac{f_2 f_3 f_{12}}{f_1 f_4}$	9312	1881	1881	1881	1881	9305	1	0	0	0
148	$\frac{f_1^2 f_2 f_3^3 f_{12}}{f_4^2 f_5 f_6^3}$	1681	1681	1681	1681	8266	2	0	0	0	0
149	$\frac{f_2^2 f_3 f_{12}}{f_1 f_4 f_6^2}$	8288	1653	1653	1653	1653	8282	1	0	0	0
150	$\frac{f_2^5 f_3^2 f_{12}}{f_1^2 f_4^2 f_6^2}$	10639	2122	2122	2122	2122	10634	2	0	0	0
151	$\frac{f_4^4 f_3^2 f_{12}}{f_1^2 f_4 f_6^2}$	9593	2024	2024	2024	2024	9589	0	0	0	0
152	$\frac{f_1^2 f_2^8 f_3 f_{12}}{f_4^3 f_5 f_6^2}$	1619	1619	1619	1619	8371	1	0	0	0	0
153	$\frac{f_3 f_{12}}{f_6}$	14910	2977	2977	2977	2977	14900	4	0	0	0
154	$\frac{f_2^4 f_{12}}{f_4^2 f_6}$	12633	2686	2686	2686	2686	12625	3	0	0	0
155	$\frac{f_2^2 f_{12}}{f_4 f_6}$	14887	2993	2993	2993	2993	14878	7	0	0	0
156	$\frac{f_3^2 f_{12}}{f_4 f_6}$	11576	3362	3362	3362	3362	11569	3	1	0	0
157	$\frac{f_4 f_6}{f_2 f_3 f_{12}}$	11968	5286	5286	5286	5286	11961	33	0	0	0
158	$\frac{f_2 f_3 f_{12}}{f_1 f_4 f_6}$	14868	3229	3229	3229	3229	14858	1	0	0	0
159	$\frac{f_2^3 f_3 f_{12}}{f_1 f_4 f_6}$	8437	1717	1717	1717	1717	8430	0	0	0	0
160	$\frac{f_1 f_2 f_6 f_{12}}{f_3 f_4}$	6966	1436	1436	1436	1436	6961	0	0	0	0

161	$\frac{f_2 f_4^2 f_{12}}{f_6 f_8}$	11530	2363	2363	2363	2363	11524	2	0	0	0
162	$\frac{f_2 f_3 f_4^2 f_{12}}{f_1 f_6 f_8}$	8437	1654	1654	1654	1654	8430	2	0	0	0
163	$\frac{f_2^6 f_3 f_8 f_{12}}{f_1^2 f_4^3 f_6^2}$	9183	1864	1864	1864	1864	9179	1	0	0	0
164	$\frac{f_2^2 f_8 f_{12}}{f_4 f_6}$	11380	2295	2295	2295	2295	11371	0	0	0	0
165	$\frac{f_2 f_3 f_5^2 f_{12}}{f_1 f_6 f_{10}}$	8665	3199	3199	3199	3199	8657	5	0	0	0
166	$\frac{f_2^4 f_3 f_{12}^2}{f_1 f_4^2 f_6^2}$	6966	1758	1758	1758	1758	6961	1	0	0	0
167	$\frac{f_8 f_{12}^2}{f_6}$	12345	2491	2491	2491	2491	12338	5	0	0	0
168	f_{13}^2	14457	2889	2889	2889	2889	14447	9	0	0	0
169	f_{14}	14957	3097	3097	3097	3097	14947	11	0	0	0
170	$\frac{f_3^3}{f_4 f_{16}}$	14910	3029	3029	3029	3029	14900	8	0	0	0
171	$\frac{f_4 f_6 f_8^3}{f_2 f_{12} f_{16}^2}$	11326	2212	2212	2212	2212	11318	4	0	0	0
172	$\frac{f_1 f_4 f_6^2 f_{10}}{f_2^2 f_3 f_5^2 f_{20}^2}$	8665	7137	7137	7137	7137	8657	2	0	0	0
173	$\frac{f_3^3 f_{10}}{f_1 f_4 f_5 f_{20}}$	12168	3770	3770	3770	3770	12161	9	0	0	0
174	$\frac{f_1 f_8 f_{12}^2}{f_3 f_4 f_6 f_{24}^3}$	8288	1614	1614	1614	1614	8282	0	0	0	0
175	$\frac{f_2^3 f_3 f_8 f_{12}^3}{f_1 f_4^3 f_6^4 f_{24}^3}$	8288	1654	1654	1654	1654	8282	0	0	0	0
176	$\frac{f_2 f_8 f_{12}^2}{f_4^2 f_6^3 f_{24}^3}$	11576	2308	2308	2308	2308	11569	3	0	0	0
177	$\frac{f_2 f_8^2 f_{12}}{f_4^2 f_6 f_{24}^2}$	10867	2258	2258	2258	2258	10859	1	0	0	0
178	$\frac{f_8 f_{12}^5}{f_6^2 f_{24}^2}$	13443	2646	2646	2646	2646	13434	7	0	0	0
179	$\frac{f_2 f_8 f_{12}^5}{f_4^2 f_6 f_{24}^2}$	11530	2251	2251	2251	2251	11524	2	0	0	0
180	$\frac{f_2^2 f_8 f_{12}^2}{f_4^3 f_6^3 f_{24}^2}$	11523	2336	2336	2336	2336	11516	0	1	0	0
181	$\frac{f_1 f_6^2 f_8^3}{f_2 f_3 f_4 f_{24}}$	11230	2194	2194	2194	2194	11223	0	0	0	0
182	$\frac{f_4 f_6 f_8 f_{12}}{f_2 f_{24}}$	10867	2245	2245	2245	2245	10859	2	0	0	0
183	$\frac{f_1 f_6^2 f_8^2 f_{12}}{f_2 f_3 f_4 f_{24}}$	11415	2306	2306	2306	2306	11409	0	0	0	0
184	$\frac{f_2^2 f_3 f_8^3 f_{12}}{f_1 f_4^2 f_6 f_{24}}$	11230	2237	2237	2237	2237	11223	1	1	0	0
185	$\frac{f_4 f_6 f_{12}^2}{f_2 f_{24}}$	11530	2267	2267	2267	2267	11524	3	0	0	0
186	$\frac{f_1 f_4 f_6 f_{12}^2}{f_2 f_3 f_{24}}$	10841	2153	2153	2153	2153	10836	1	0	0	0
187	$\frac{f_1 f_4^3 f_6 f_{12}^2}{f_2^2 f_3 f_8 f_{24}}$	8266	1753	1753	1753	1753	8260	0	0	0	0
188	$\frac{f_8 f_{12}^2}{f_4 f_{24}}$	14910	3119	3119	3119	3119	14900	4	0	0	0
189	$\frac{f_2 f_8 f_{12}}{f_6 f_{24}}$	11326	2288	2288	2288	2288	11318	1	0	0	0
190	$\frac{f_2^2 f_8^2 f_{12}}{f_4^2 f_6 f_{24}}$	12633	2497	2497	2497	2497	12625	4	0	0	0
191	$\frac{f_2^2 f_3 f_8^2 f_{12}^2}{f_1 f_4^2 f_6 f_{24}}$	11415	2323	2323	2323	2323	11409	3	0	0	0
192	$\frac{f_2^2 f_3 f_{12}^3}{f_1 f_6^2 f_{24}}$	10841	2263	2263	2263	2263	10836	1	0	0	0
193	$\frac{f_{12}^3}{f_8 f_{24}}$	14928	3233	3233	3233	3233	14919	6	0	0	0
194	$\frac{f_1 f_{12}}{f_2 f_6 f_{24}}$	10841	2165	2165	2165	2165	10836	3	1	0	0
195	$\frac{f_2 f_{12}}{f_6 f_{24}}$	11523	2476	2476	2476	2476	11516	2	0	0	0

196	$\frac{f_2^5 f_{12}^3}{f_1^2 f_4^2 f_6 f_{24}}$	10841	2229	2229	2229	2229	10836	2	0	0	0
197	$\frac{f_1 f_6 f_{12}^3}{f_2 f_3 f_{24}}$	9312	1865	1865	1865	1865	9305	0	0	0	0
198	$\frac{f_2 f_3 f_4^2 f_{12}^3}{f_1 f_8^2 f_8 f_{24}}$	8266	1684	1684	1684	1684	8260	2	0	0	0
199	$\frac{f_4^3 f_{12}^3}{f_2 f_6 f_8 f_{24}}$	10737	2346	2346	2346	2346	10731	3	0	0	0
200	$\frac{f_8^3 f_{12}^3}{f_6 f_{24}^4}$	12079	2429	2429	2429	2429	12069	2	0	0	0
201	$\frac{f_2^2 f_3 f_{12}^4}{f_1 f_4 f_6^2 f_{24}}$	9312	1840	1840	1840	1840	9305	2	0	0	0
202	$\frac{f_2^4 f_8^2 f_{12}^2}{f_4^2 f_6 f_{16} f_{24}}$	11326	2272	2272	2272	2272	11318	3	0	0	0
203	$\frac{f_1^2 f_2^2 f_3 f_{24}}{f_4 f_5 f_6^2 f_8}$	1664	1664	1664	1664	8553	0	0	0	0	0
204	$\frac{f_1 f_4^3 f_6^2 f_{24}}{f_2^3 f_3^3 f_8 f_{12}^4}$	8553	1744	1744	1744	1744	8547	0	0	0	0
205	$\frac{f_1 f_4^3 f_6^2 f_{24}}{f_2^4 f_3 f_8^3 f_{12}^3}$	8288	1592	1592	1592	1592	8282	2	0	0	0
206	$\frac{f_4^6 f_8^3 f_{24}}{f_2^3 f_8^2 f_{12}^3}$	11523	2239	2239	2239	2239	11516	2	0	0	0
207	$\frac{f_4^{10} f_6 f_{24}}{f_2^4 f_8^4 f_{12}^2}$	12633	2529	2529	2529	2529	12625	1	0	0	0
208	$\frac{f_3 f_4^4 f_{24}}{f_1 f_2 f_8^3 f_{12}^2}$	8288	1696	1696	1696	1696	8282	1	0	0	0
209	$\frac{f_4^8 f_6 f_{24}}{f_2^3 f_8^3 f_{12}^2}$	11576	2254	2254	2254	2254	11569	3	0	0	0
210	$\frac{f_4^5 f_6 f_{24}}{f_2^2 f_8^2 f_{12}^2}$	14887	3463	3463	3463	3463	14878	3	0	0	0
211	$\frac{f_4^5 f_6 f_{24}}{f_2^2 f_8^2 f_{12}^2}$	11530	2314	2314	2314	2314	11524	1	0	0	0
212	$\frac{f_1 f_4^4 f_6^3 f_{24}}{f_2^3 f_3 f_8^2 f_{12}^2}$	6966	1416	1416	1416	1416	6961	0	0	0	0
213	$\frac{f_4^5 f_6 f_{24}}{f_2^2 f_8 f_{12}^2}$	11380	2285	2285	2285	2285	11371	1	0	0	0
214	$\frac{f_2^2 f_4^4 f_6^3 f_{24}}{f_1 f_3 f_8 f_{12}^2}$	8534	1767	1767	1767	1767	8527	1	0	0	0
215	$\frac{f_3 f_4^4 f_{24}}{f_1 f_8^2 f_{12}}$	6966	1401	1401	1401	1401	6961	1	0	0	0
216	$\frac{f_1 f_3 f_4^2 f_{24}}{f_2 f_8 f_{12}}$	8534	1637	1637	1637	1637	8527	1	0	0	0
217	$\frac{f_3 f_4^4 f_{24}}{f_1 f_6 f_8 f_{12}}$	8553	1792	1792	1792	1792	8547	0	0	0	0
218	$\frac{f_4^2 f_5 f_6 f_{24}}{f_1^2 f_2^2 f_3 f_8 f_{12}}$	1667	8553	1667	1667	1667	0	8547	1	0	0
219	$\frac{f_6 f_8 f_{24}}{f_{12}}$	12345	2426	2426	2426	2426	12338	2	0	0	0
220	f_{26}	14971	3064	3064	3064	3064	14961	8	0	0	0
221	$\frac{f_{14}^6}{f_7^2 f_{28}^2}$	14037	2684	2684	2684	2684	14028	4	1	0	0
222	$\frac{f_2^2 f_{14}^3}{f_1 f_4 f_7 f_{28}}$	12528	3514	3514	3514	3514	12522	1	0	0	0
223	$\frac{f_{16}^3}{f_8 f_{32}}$	14939	3258	3258	3258	3258	14929	21	0	0	0
224	$\frac{f_2^2 f_3 f_{16}^3}{f_1 f_6 f_8 f_{32}}$	12144	2412	2412	2412	2412	12138	3	0	0	0
225	$\frac{f_1 f_4 f_6^2 f_{16}^3}{f_2 f_3 f_8 f_{12} f_{32}}$	12144	2565	2565	2565	2565	12138	1	0	0	0
226	$\frac{f_{12} f_{18}}{f_6 f_{36}}$	14928	3048	3048	3048	3048	14919	2	0	0	0
227	$\frac{f_{18}^3}{f_9 f_{36}}$	14943	3142	3142	3142	3142	14933	12	0	0	0
228	$\frac{f_1 f_{18}}{f_9 f_{36}}$	8699	1768	1768	1768	1768	8692	1	0	0	0
229	$\frac{f_2^2 f_{18}^3}{f_1 f_4 f_9 f_{36}}$	8688	1720	1720	1720	1720	8681	0	0	0	0

230	$\frac{f_3^2 f_{18}^3}{f_6 f_9 f_{36}}$	13895	2742	2742	2742	2742	13887	2	0	0	0
231	$\frac{f_2 f_3^2 f_{18}^3}{f_1 f_6 f_9 f_{36}}$	8725	1766	1766	1766	1766	8718	6	0	0	0
232	$\frac{f_3 f_4 f_{18}^3}{f_6 f_9 f_{36}}$	9307	1889	1889	1889	1889	9298	3	0	0	0
233	$\frac{f_1 f_4 f_6^5 f_{18}^3}{f_2^2 f_3^2 f_9 f_{12} f_{36}}$	8685	1773	1773	1773	1773	8678	2	0	0	0
234	$\frac{f_{20}^3}{f_{10} f_{40}}$	14947	3037	3037	3037	3037	14937	21	0	0	0
235	$\frac{f_2^2 f_{20}^3}{f_2 f_{10} f_{40}}$	11552	2272	2272	2272	2272	11544	3	0	0	0
236	$\frac{f_2^3 f_{20}^3}{f_1^2 f_4^2 f_{10} f_{40}}$	11552	2346	2346	2346	2346	11544	2	0	0	0
237	$\frac{f_4 f_{16} f_{24}^5}{f_8^2 f_{12}^2 f_{48}^2}$	14910	2988	2988	2988	2988	14900	9	0	0	0
238	$\frac{f_{26}^3}{f_{13}^2 f_{52}^2}$	14457	2886	2886	2886	2886	14447	1	0	0	0
239	$\frac{f_{14}^4 f_{28}}{f_7^2 f_{56}}$	14037	2839	2839	2839	2839	14028	1	1	0	0
240	$\frac{f_7^2 f_{28}^3}{f_{14}^3 f_{56}}$	14037	2783	2783	2783	2783	14028	0	0	0	0
241	$\frac{f_{28}^3}{f_{14} f_{56}}$	14957	2948	2948	2948	2948	14947	1	0	0	0
242	$\frac{f_{20} f_{30}}{f_{10} f_{60}}$	14947	2949	2949	2949	2949	14937	3	0	0	0
243	$\frac{f_6 f_{24} f_{36}^5}{f_{12}^2 f_{18}^2 f_{72}^2}$	14928	2972	2972	2972	2972	14919	6	0	0	0
244	$\frac{f_{28} f_{42}}{f_{14} f_{84}}$	14957	3002	3002	3002	3002	14947	4	0	0	0
245	$\frac{f_{26}^2 f_{52}}{f_{13}^2 f_{104}}$	14457	2865	2865	2865	2865	14447	3	0	0	0
246	$\frac{f_{13}^2 f_{52}^3}{f_{26}^2 f_{104}}$	14457	2870	2870	2870	2870	14447	1	0	0	0
247	$\frac{f_{52}^3}{f_{26} f_{104}}$	14971	2918	2918	2918	2918	14961	3	0	0	0
248	$\frac{f_{10} f_{40} f_{60}^5}{f_{20}^2 f_{30}^2 f_{120}^2}$	14947	3067	3067	3067	3067	14937	18	0	0	0
249	$\frac{f_{52}^2 f_{78}}{f_{26} f_{156}}$	14971	2890	2890	2890	2890	14961	3	0	0	0
250	$\frac{f_{14} f_{56} f_{84}^5}{f_{28}^2 f_{42}^2 f_{168}^2}$	14957	2960	2960	2960	2960	14947	4	0	0	0
251	$\frac{f_{26} f_{104} f_{156}^5}{f_{52}^2 f_{78}^2 f_{312}^2}$	14971	2961	2961	2961	2961	14961	2	0	0	0
252	$\frac{f_4^2 f_5^2}{f_1 f_2}$	1257	1257	1257	1257	1257	0	135	0	0	0
253	$f_1 f_3^4 f_5$	1188	1188	1188	1188	1188	477	0	1	0	0
254	$\frac{f_1^2 f_2^5 f_5}{f_4^2}$	1258	1258	1258	1258	1258	134	0	0	0	0

Table 2: The count of zero coefficients modulo 25 (columns 3 -7) and the count of zero coefficients (columns 8 - 12) in the first 15000 terms in the series expansion of $F(q)(f_1^5/f_5)^j$, $0 \leq j \leq 4$. This time only 3 of the 5 eta quotients have identically vanishing coefficients, modulo 25.

n	$F(q)$	0	1	2	3	4	0	1	2	3	4	C_N	N
1	$f_1 f_2$	10505	7436	7436	7436	7437	10500	58	0	0	0	10441	5076
2	f_2^2	11926	7572	7572	7573	7572	11918	0	3	0	0	4761	2254
3	f_3^2	12881	3176	3177	3176	3176	12874	474	0	0	0	7141	1421
4	$\frac{f_4^4}{f_1 f_4}$	10505	2929	2929	2930	2929	10500	5	0	0	0	10441	1951
5	$\frac{f_2^2 f_3}{f_1 f_4}$	10815	2101	2101	2101	2102	10808	2	0	0	0	3571	470
6	$\frac{f_1 f_3 f_4}{f_2}$	10639	2030	2031	2030	2030	10634	3	0	0	0	7141	929

7	f_4^2	13383	5679	5680	5679	5679	13375	442	0	0	0	9521	3496
8	$\frac{f_1 f_2^{38}}{f_4^4 f_5^3}$	1596	1596	7891	1597	1596	0	0	0	0	0	7141	692
9	$\frac{f_5^3}{f_1 f_2^4}$	6120	6119	6119	7891	6119	0	0	5661	7887	0	7141	2780
10	$\frac{f_2^3 f_3}{f_1^2 f_6}$	10505	2317	2317	2318	2317	10500	1	0	0	0	10441	1581
11	$\frac{f_1 f_3^2}{f_2 f_6}$	11652	3445	3447	3445	3445	11645	0	0	0	0	7204	1561
12	$\frac{f_3^3 f_3}{f_1 f_4 f_6}$	9594	1884	1884	1884	1885	9588	0	0	0	0	5431	653
13	$\frac{f_2 f_3 f_4}{f_1 f_6}$	8390	1630	1630	1630	1631	8385	2	0	0	0	5951	631
14	$\frac{f_2^2 f_6}{f_1}$	11853	2476	2476	2476	2477	11845	5	0	0	0	5491	854
15	$\frac{f_2^2 f_6}{f_1}$	13339	2759	2759	2760	2759	13331	3	0	0	0	3661	632
16	$\frac{f_4}{f_1 f_4 f_6}$	11853	2404	2403	2403	2403	11845	2	0	0	0	5491	850
17	$\frac{f_2}{f_5 f_6}$	2221	10505	2222	2221	2221	0	10500	2	0	0	10441	1514
18	$\frac{f_1 f_2^2 f_3}{f_4 f_5 f_6}$	2338	11435	2338	2339	2338	0	11428	0	0	0	8282	1238
19	$\frac{f_1 f_6}{f_2 f_3}$	11853	3088	3088	3089	3088	11845	3	0	0	0	5491	1027
20	$\frac{f_3^2 f_6}{f_1 f_3 f_4}$	8124	1687	1686	1686	1686	8116	1	0	0	0	11901	1337
21	$\frac{f_2 f_6}{f_3^2 f_4}$	10855	2154	2154	2155	2154	10845	0	0	0	0	9761	1401
22	$\frac{f_1 f_4 f_6^4}{f_2^2 f_3}$	7688	1535	1535	1535	1536	7685	1	0	0	0	11591	1177
23	$\frac{f_2^2 f_6}{f_8}$	12812	2570	2570	2570	2571	12805	1	0	0	0	7141	1215
24	$\frac{f_1^2 f_8}{f_2}$	12924	3434	3435	3434	3434	12917	112	0	0	0	9521	2098
25	$\frac{f_4^4 f_8}{f_2}$	11864	2493	2493	2494	2493	11855	5	0	0	0	6203	985
26	$\frac{f_2 f_8^2}{f_4}$	11864	2704	2704	2705	2704	11855	12	0	0	0	6203	1038
27	$\frac{f_3^2 f_9}{f_6}$	13855	2778	2778	2778	2779	13846	1	0	0	0	5491	971
28	$\frac{f_2 f_5^3}{f_1 f_{10}}$	7891	6119	6120	6119	6119	7887	5661	0	0	0	7141	2780
29	$\frac{f_1 f_4 f_6^7}{f_2^3 f_3^3 f_{12}^2}$	7127	1401	1402	1401	1401	7124	1	0	0	0	5491	491
30	$\frac{f_1^2 f_6^4}{f_2 f_3^2 f_{12}}$	10639	2150	2151	2150	2150	10634	1	0	0	0	7141	1026
31	$\frac{f_3^3 f_6^3}{f_2^2 f_8 f_{12}}$	12733	2537	2537	2538	2537	12726	4	1	0	0	5257	862
32	$\frac{f_4 f_6^2 f_8}{f_2 f_{12}}$	11564	2258	2258	2258	2259	11556	2	0	0	0	11901	1768
33	$\frac{f_2^3 f_3 f_{12}}{f_1 f_4 f_6^5}$	7127	1399	1399	1400	1399	7124	0	0	0	0	5491	494
34	$\frac{f_1 f_3 f_{12}}{f_6}$	8124	1626	1627	1626	1626	8116	6	0	0	0	11901	1264
35	$\frac{f_2^2 f_3 f_{12}}{f_1 f_4 f_6}$	11853	2318	2318	2318	2319	11845	4	0	0	0	5491	815
36	$\frac{f_1 f_6 f_{12}^2}{f_2 f_3}$	8877	1814	1815	1814	1814	8874	1	0	0	0	7827	944
37	$\frac{f_2^2 f_3 f_{12}^3}{f_1 f_4 f_6^2}$	8877	1824	1823	1823	1823	8874	1	0	0	0	7827	974
38	$\frac{f_6^6}{f_4^2 f_{16}^2}$	13383	3700	3700	3700	3701	13375	109	0	0	0	9521	2269
39	$\frac{f_4^4 f_8}{f_2^2 f_{16}}$	11926	2326	2325	2325	2325	11918	5	0	0	0	4761	732
40	$\frac{f_2^2 f_8^3}{f_4^2 f_{16}}$	11926	2308	2307	2307	2307	11918	3	0	0	0	4761	718
41	$\frac{f_1 f_4 f_{10}^8}{f_2^2 f_3^3 f_{20}^3}$	7891	1597	1596	1596	1596	7887	193	0	0	0	7141	692
42	$\frac{f_{10}}{f_2^2 f_{20}^2}$	13693	2745	2745	2746	2745	13684	15	0	0	0	11901	2151
43	$\frac{f_2^3 f_8 f_{12}^2}{f_4^3 f_3^3 f_{24}^3}$	12733	2650	2651	2650	2650	12726	3	0	0	0	5257	911
44	$\frac{f_8 f_{12}}{f_4 f_6 f_{24}^2}$	12812	2565	2565	2565	2566	12805	0	0	0	0	7141	1219
45	$\frac{f_2 f_8^2 f_{12}^2}{f_4^4 f_6^2 f_{24}^2}$	11564	2328	2328	2329	2328	11556	2	0	0	0	11901	1850
46	$\frac{f_6 f_{12}}{f_3^2 f_{24}}$	12881	2603	2603	2604	2603	12874	1	0	0	0	7141	1211
47	$\frac{f_6 f_8^2 f_{12}}{f_4 f_{24}}$	12812	2568	2567	2567	2567	12805	1	0	0	0	7141	1194

48	$\frac{f_3^2 f_{12}^3}{f_6^2 f_{24}^3}$	12881	2547	2547	2547	2548	12874	0	0	0	0	7141	1184
49	$\frac{f_4^2 f_{12}^3}{f_6 f_8 f_{24}^3}$	12812	2611	2611	2611	2612	12805	1	0	0	0	7141	1176
50	$\frac{f_8^4 f_{16}^3}{f_4^3 f_{32}^3}$	13383	2918	2917	2917	2917	13375	115	0	0	0	9521	1856
51	$\frac{f_4^2 f_{16}^3}{f_8^2 f_{32}^3}$	13383	3493	3492	3492	3492	13375	225	0	0	0	9521	2121
52	$\frac{f_1^2 f_{16}^3}{f_2 f_8 f_{32}^3}$	12894	2834	2833	2833	2833	12887	114	0	0	0	9521	1780
53	$\frac{f_6^2 f_8^3}{f_3 f_9 f_{12} f_{36}^3}$	13855	2749	2749	2749	2750	13846	2	0	0	0	5491	974
54	$\frac{f_3^2 f_4 f_{12} f_{18}^3}{f_1 f_6^3 f_9^2 f_{36}^3}$	7127	1373	1373	1374	1373	7124	0	0	0	0	5491	495
55	$\frac{f_1 f_4^3 f_6^3 f_9^2 f_{36}^3}{f_2^3 f_4^4 f_{12}^3 f_{18}^3}$	7127	1400	1400	1400	1401	7124	0	0	0	0	5491	511
56	$\frac{f_{10}^2 f_{20}^3}{f_5^2 f_{40}^3}$	13693	2646	2645	2645	2645	13684	3	0	0	0	11901	2107
57	$\frac{f_2^2 f_{20}^3}{f_{10}^2 f_{40}^3}$	13693	2708	2707	2707	2707	13684	18	1	0	0	11901	2134
58	$\frac{f_8^2 f_{10}^3}{f_2^2 f_8^4 f_{10} f_{40}^3}$	13070	2977	2978	2977	2977	13060	8	0	0	0	11901	2342

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SCHOOL OF MATHEMATICAL AND STATISTICAL SCIENCES, UNIVERSITY OF TEXAS RIO GRANDE VALLEY, EDINBURG, TEXAS 78539, USA

Email address: timothy.huber@utrgv.edu

MATHEMATICS DEPARTMENT, 25 UNIVERSITY AVENUE, WEST CHESTER UNIVERSITY, WEST CHESTER, PA 19383

Email address: jmclaughlin2@wcupa.edu

SCHOOL OF MATHEMATICS (ZHUHAI), SUN YAT-SEN UNIVERSITY, ZHUHAI 519082, GUANGDONG, PEOPLE'S REPUBLIC OF CHINA

Email address: yedx3@mail.sysu.edu.cn