When modeling systems made up of two materials with different thermodynamic properties, a physical interface can be introduced to account for the border where the materials meet. This interface separates our model’s standard grid into two regions, each with its unique physical properties. At these interfaces, boundary conditions can be imposed to represent the difference in heat and in heat flux between the different materials so that their interaction may be modeled accurately. Because standard finite difference methods are inadequate to deal with interfaces, a Matched Interface and Boundary (MIB) technique is investigated in this work to solve the heat equation with interfaces. MIB techniques are powerful tools used to solve partial differential equations due to their efficiency and stability [4]. Without loss of generality, this work will solve 1-dimensional interface problems to demonstrate the accuracy and computational efficiency of this method, which will create a linear system of equations to be solved at each step in time throughout the duration of the model.

**Governing Model of the Interface Problem**

The heat equation for two distinct domains by an interface \( \Gamma \) is given by the following:

\[
u_{\Omega} = \nabla (\alpha \nabla u) + f \quad \Omega = \Omega^+ \cup \Omega^- \quad \Gamma = \Omega^+ \cap \Omega^-
\]

where \( \alpha(x,t) \) is the temperature at a given time, \( \alpha \) is the diffusion coefficient, \( \Omega \) is a square domain containing \( \Omega^+ \) and \( \Omega^- \) and \( f \) is the source term. The jump conditions are as follows

\[ u|_{=\Omega} = u^+ - u^- = \phi(s,t), \quad [u_{\alpha}] = \alpha^+ \frac{\partial u^+}{\partial n} - \alpha^- \frac{\partial u^-}{\partial n} = \psi(s,t) \quad (2) \]

**Demonstration of Methods in 2 Dimensions Continued**

- **Spatial Discretization: Matched Interface and Boundary Method (continued)**
  - Decomposition of the jump conditions to \( x \) and \( y \) directions
    \[
    \frac{\partial}{\partial t} u^+ = \cos(\theta) \frac{\partial}{\partial x} u^+ + \sin(\theta) \frac{\partial}{\partial y} u^+, \quad \frac{\partial}{\partial t} u^- = -\sin(\theta) \frac{\partial}{\partial x} u^- + \cos(\theta) \frac{\partial}{\partial y} u^- \quad (7)
    \]
    - Here, however, we can see an obvious coupling of the \( x \) and \( y \) directions incorporated into the flux jump conditions
      \[
      \cos(\theta) [u_{\alpha}] + \sin(\theta) [\psi] = \psi \quad (8)
      \]
    - Giving us the derived jump conditions
      \[
      [u_{\alpha}] = \psi \cos \theta - \sin \theta (\alpha^+ - \alpha^-) u_{\psi}^+ - \sin \theta [\psi\alpha^-] = \psi \quad (9)
      \]
    - To modify the central difference formula in the MIB scheme at irregular nodes immediately juxtaposed with the interface \( \Gamma \) in the MIB scheme.
      \[
      \delta_y u_{ij}^k = \frac{1}{4} (u_{ij}^{k+1} - 2u_{ij}^{k+2} + u_{ij}^{k+3}), \quad \delta_y u_{ij}^k = \frac{1}{4} (u_{ij}^{k+1} - 2u_{ij}^{k+2} + u_{ij}^{k+3})
      \]
    where \( u_{ij}^k \) is considered for both the \( x \) and \( y \) directions independently and \( u_{ij}^k \) and \( u_{ij}^k \) are additional “fictitious values” that represent the approximation of nodes on the opposite side of the interface.

**Numerical Experiment in 1 Dimension**

**Example 1**

The analytical solution is given as

\[ u(x,t) = \left\{ \begin{array}{ll}
\cos(x) \sin(t) & \Omega^+ \\
\Omega^-
\end{array} \right. \]

With the jump conditions

\[ \phi = \cos(x) \sin(t) - \sin(x) \cos(t) \quad (10) \]

\[ \psi = -10 \sin(x) \sin(t) - \cos(x) \cos(t) \quad (11) \]

**Example 2**

The analytical solution is given as

\[ u(x,t) = \left\{ \begin{array}{ll}
\cos(x) \sin(t) & \Omega^+ \\
\Omega^-
\end{array} \right. \]

With the jump conditions

\[ \phi = \cos(x) \sin(t) - \cos(x) \cos(t) \quad (12) \]

\[ \psi = -10 \sin(x) \sin(t) - \cos(x) \cos(t) \quad (13) \]

Since the method is shown to be accurate for the example cases given, we can now use this method to get solutions to problems where the analytical solution is unknown. Further improvements can be made going forward from the current investigation. To increase the number of spatial dimensions used, an Alternating Direction Implicit method can be used in conjunction with the MIB method. Making these improvements allows for more complex models to be used in the future, which can be used in a number of applications.

**Conclusion**